Green Coins

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Abstract

We propose a novel macrofinance model for the EU area comprising climate damages, an Emission Trading System, and credit markets with (1) a bias for 'net-zero' brown firms, and (2) opaque voluntary carbon credit markets. Our model suggests that the implied allocation is far from the first-best. Relevant welfare gains can be obtained in a setting in which a Green Coin Central Bank (GCCB) runs a transparent blockchain in which coins are mined after a decentralized verification of private offsets, and the GCCB manages green coin prices through open market operations.

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1 Introduction

Emission regulation is a priority that poses many challenges and prompts several key macroeconomic questions. How can we let emissions regulation coexist with decentralized private markets? How can we prevent fraudulent emissions miscounting due to asymmetric information? In addition, how can we design policy tools that promptly react to economic- and climate-related conditions? To address these questions, we need a model of the interactions of public emission trading systems (ETS), private carbon credit markets, and carbon-sensitive credit markets. In addition, we advocate for a novel regulation mechanism that takes advantage of blockchain technologies in order to develop a digital green coin market in which an authority can alter the cost of emissions in real time. More broadly, this study intends to promote a novel *"Green MacroFinTech"* approach to emissions regulation.

Specifically, we take seriously corporate frictions and study an economy in which a brown sector finances its investments with a mix of equity and corporate debt. Corporate debt is preferred to equity because of a tax shield and is limited by a borrowing constraint. This constraint is tied to the net emissions reported by brown firms, enabling firms with lower net emissions to access more credit. Net emissions are computed as gross emissions minus declared offsets. In addition, following of the European ETS, we assume that the brown sector faces an emission constraint, i.e., by regulation, corporate net emissions must be matched by emissions allowances whose supply is decided by a central authority that allocates them once a year.

In this setting, the price of allowances is related to the marginal damage to added value that firms would face if they are unable to match additional production-related emission with allowances. Note that the brown sector has a twofold incentive to buy privately issued carbon credits. First, these credits reduce net emissions and enable firms to get more credit by appearing as 'net-zero' firms. Second, depending on the ETS rules, these credits can lower the need of allowances and relax the regulation constraint.

The economy comprises a second sector that uses green assets, i.e., all assets that con-

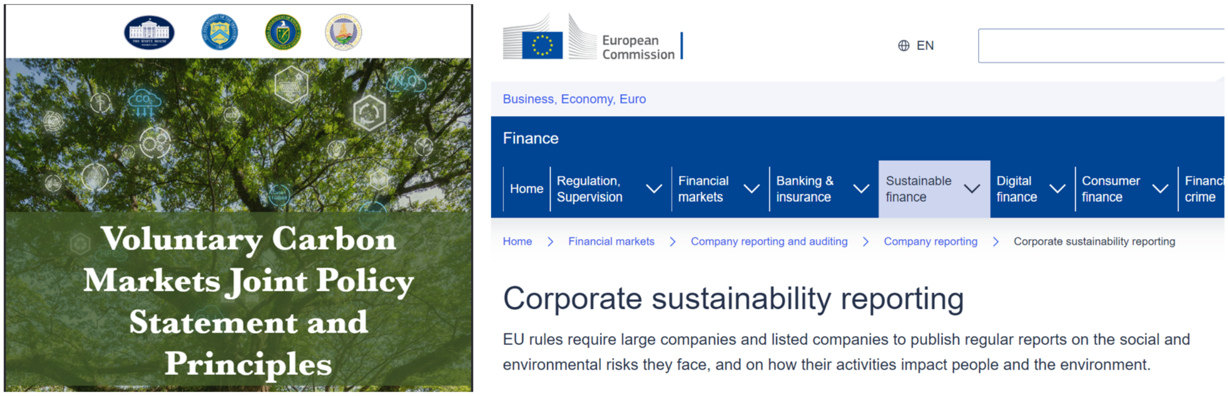

Fig. 1: Policy News in the USA and in the EU

Notes: The left panel refers to the May 2024 White House joint statement about voluntary carbon markets joint policy. The right panel refers to the reporting requirements that the EU has introduced following the Corporate Sustainability Reporting Directive (CSRD, 2022/2464/EU).

tribute to either offsetting or sequestering CO_2 from the atmosphere. For the sake of simplicity, the green sector does not produce a final good. Rather, it provides a valuable service to the brown sector by reducing CO_2 and issuing private carbon credits accordingly. In order to capture the opaqueness of this market, we assume the existence of a non-negative wedge between declared offsets and real offsets. This is a key concern identified by consumer advocates, specialized press, and regulators. Indeed, in the US, the White House has highlighted this issue in the White House joint statement about voluntary carbon markets joint policy. In the EU, the ETS regulation has been modified several times in the last few years in order to avoid an inappropriate use of private carbon credits. Moreover, in the aftermath of the European 2019 'Green Deal', the European Commission has approved a very relevant Corporate Sustainability Reporting Directive (CSRD, 2022/2464/EU).

After compiling a novel dataset that merges macroeconomic aggregate data with prices on emission allowances, futures on private carbon credits, emissions, and green investments for the EU economy, we calibrate this model and show that the model captures important features of macro quantities both in the brown and green sector as well as carbon prices and asset returns in these two sectors. In a second step, we consider a second setting with

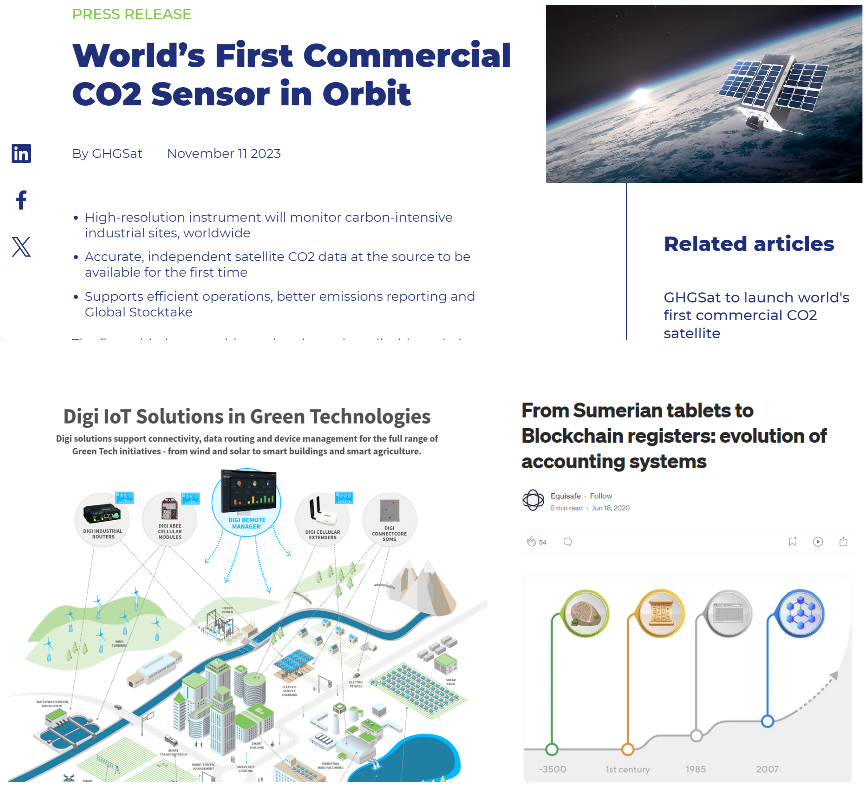

Fig. 2: New Monitoring Technologies

Notes: The top panel refers to new satellite technology that identifies CO_2 emission with great detail offered by GHGSat. The bottom panel refers to the interplay of IoT and Blockchain.

a radically different way to deal with emissions based on the decentralized monetization of emissions. Our results suggest that this setting could produce sizable welfare gains.

Specifically, we propose a simplified structure in which both self-certified carbon credits and auctions of emission allowances are not present. We assume that the following protocol is in place. There is a blockchain that is run and monitored by a Green Coin Central Bank (GCCB, henceforth). A coin is mined by the private sector every time a unit of carbon offset is produced, and it is retired at the end of the period. Thanks to the new technological paradigm called 'Internet of Things' (IoT), these actions can be verified using either location services and barcodes or smart chips. As a result, the supply of green coins tracks in real-time the supply of effective offsets. In addition, the GCCB is assumed to have access to a GHG Satellite technology that enables superior monitoring on ex-post realized offsets. In short, this is a setting with real-time full information on emissions and offsets (see, for example, figure 2).

The GCCB controls the supply of green coins available to the public by running regular open market operations. In this setting, green coins are valuable to the private sector because they are the only assets that provide certified offsets and help firms meeting regulatory emission limits. By tightening or loosing the green coin policy, the GCCB can better steer the economy toward the first best. Although not directly modeled in our study, we believe that a decentralized blockchain can enable a wide range of private actors to be included in the green market. This technology is scalable and can be accessible to both small, medium, and large firms. Most importantly, in a setting in which the GCCB commits to an implementable green policy rule that responds to both economic shocks and state variables, we find that the welfare losses from the first-best are greatly reduced.

Interestingly, the damage originated by emissions acts as a distortional taxation margin since it is proportional to output. We find that the GCCB finds it optimal to implement an 'emission smoothing' policy, in the spirit of what observed in the fiscal policy literature. Exogenous shocks that immediately increase emissions are partially accommodated by the GCCB, meaning that this entity increases the supply of green coins available to the public by buying a smaller share of outstanding coins. As the stock of emissions increases over time, the GCCB slowly tightens the green coin market and increases its holdings of green coins. The welfare benefits can be as large as 6.5% of time-zero first-best consumption.

Related literature. Our policy focus is novel and hence there is a limited number of manuscripts directly related to our research. What follows is not a comprehensive review of the literature, rather it highlights a subset of key contributions. Bustamente and Zucchi (2022) focuses on a firm subject to emission constraint and enabled to buy private Verified Carbon Credits (VCCs). Our research differs from this work in many crucial-and-novel dimensions. In particular, we (i) account for the opaqueness of VCCs by considering a wedge between declared emission reductions and effective ones; (ii) have a quantitative focus; (iii) use new *EU* data; (iii) price carbon emissions in a macrofinance integrated assessment model, and (iv) propose a totally new, policy-relevant, and implementable framework based on green coins. We believe that tokenization can substantially improve emission management and resolve some of the inefficiencies highlighted in Borri, Liu, Tsyvinski, and Wu (2024).

Several modeling assumptions come from the Financial Economics of Climate literature (among others, see Giglio, Kelly, and Stroebel (2021)). We differ for our "Green MacroFinTech" approach that focuses on regulation in models that replicate both quantities and asset prices.

The interplay between macroeconomic aggregates and asset prices has been the main object of interest of the macro-finance literature (see, among many others, Ai (2007); Belo, Bazdresch, and Lin (2014); Corhay, Kung, and Schmid (2015)). The aggregate implications of emissions regulation and its interaction with FinTech in a macro-finance model are still to be assessed. Our research aims to fill this gap in the literature.

FinTech has recently attracted many contributions (see, among others, Biais, Capponi, Cong, Gaur, and Giesecke (2023), Niepelt (2024), Brunnermeier and Payne (2022)). This research proposal distinguishes its position by using FinTech at a macro-level in order to enhance climate change policies. To the best of our knowledge, nobody has focused on the benefits that FinTech can produce in the context of the green transition.

2 The Models

In what follows, we describe the settings that we intend to focus on. We start by describing the planner's problem. We then turn our attention to the setting that should be calibrated using EU data, i.e., a model with an ETS and an opaque market for VCCs. In the last step, we explore a setting with green coins.

2.1 The planner's problem

The household. We assume that the representative agent has Epstein and Zin (1989) recursive preferences so that she prices long-run risks, including those related to climate change:

$$U_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where C_t is the amount of output consumed. We do not include leisure in the utility function for the sake of parsimony. As a result, the supply of labor is fixed and normalized to one in both sectors. In this setting, the stochastic discount factor (SDF) is

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}}{\left[\mathbb{E}_t\, U_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}.$$

The brown sector. The brown sector produces output, Y_t, according to the following production function:

$$Y_t = (Z_t L_{bt})^{1-\alpha} K_{bt-1}^{\alpha}, \tag{1}$$

where Z, L_b, K_b, denote productivity, labor, and capital deployed in this sector, respectively. Capital evolves as follows,

$$K_{bt} = (1-\delta)K_{bt-1} + \Psi(I_{bt}/K_{bt-1})K_{bt-1}, \tag{2}$$

where Ψ denotes Jermann (1998) convex adjustment costs.

Similar to Shapiro and Metcalf (2023), the fraction of output that is not disrupted by climate damage is determined by the following function

$$\chi(s_t) = \exp(-\chi_0 \cdot s_t), \tag{3}$$

where s_t captures carbon dioxide concentration and accumulates depending on net emissions in the economy. In what follows, output net of climate-related damages is denoted by Y_t^n and determined as follows:

$$Y_t^n = \chi(s_t) \cdot Y_t. \tag{4}$$

Gross emissions produces by the brown sector is assumed to be proportional to gross output:

$$E_t = \lambda_t \cdot Y_t, \tag{5}$$

where the process that determines emissions per unit of added value is subject to the external shocks,

$$\log \lambda_t \quad = (1 - \rho_\lambda)\mu_\lambda + \rho_\lambda \log \lambda_{t-1} + \sigma_\lambda \epsilon_{\lambda,t}. \tag{6}$$

We specify this law of motion in log units so that λ_t is always non negative. As in the data, we model λ_t as a persistent process. Among other channels, external shocks capture unmodeled technology improvements as well as natural disasters (e.g., fires) and changes in energy sources policy (e.g., shifts from renewable to coal-based energy).

In addition, we allow for the negative feedback between CO_2 and output by defining aggregate productivity growth, Δz, as follows:

$$\begin{aligned}
\Delta z_{t+1} &= \mu_z + x_t + \sigma_z \epsilon_{z,t+1} \\
x_t &= \rho_x x_{t-1} - \phi_s \cdot (\log(s_{t-1}) - \log(\overline{s})) + \sigma_x \epsilon_{x,t}.
\end{aligned}$$

The specification above is common in the long-run risk literature, except for the fact that it accounts for an endogenous disruptive effect of CO_2 on expected long-run growth ($\phi_s > 0$). Thus, green assets are beneficial because not only they help relax emission constraint but also indirectly improves productivity by reducing CO_2 concentration. In what follows, we denote ϵ_z and ϵ_x as short- and long-run productivity shocks, respectively.

The green sector. The green sector does not participate to the goods market. It solely produces CO_2 offsets according to the following production function:

$$G_t = (Z_{gt} L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa}, \tag{7}$$

where capital evolves as follows,

$$K_{gt} = (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}, \tag{8}$$

and H captures convex adjustment costs as in Jermann (1998). Along the balanced growth path, productivity in this sector is cointegrated with that of the brown sector:

$$\log \frac{Z_{g,t}}{Z_t} - \overline{zc} = (1 - \phi_g) \left(\log \frac{Z_{g,t-1}}{Z_{t-1}} - \overline{zc} \right) - (1 - \phi_g)(\Delta z_t - \mu). \tag{9}$$

We model the cointegration residual so that the green sector productivity adapts slowly to innovations in the brown sector (small ϕ_g), consistent with the fact that green technology innovations arrive at a slower pace compared to innovations in the traditional brown sector (e.g., slow improvement in solar panels efficiency, slow improvement in the efficiency-emissions ratio for the production of batteries). The parameter $\overline{zc}$ accounts for differences in the levels of productivity which are to be expected given that this sector specializes on offsets, whereas the brown sector focuses on goods and services.

The climate feedback. Let NE_t denote net offsets in the economy:

$$NE_t := E_t - G_t, \tag{10}$$

we model the accumulation of CO_2 as follows:

$$s_t = (1 - \delta_s)s_{t-1} + NE_t/Z_t, \tag{11}$$

where net emissions are standardized by productivity and hence must be interpreted as relative to the size of the economy and δ_s captures the fraction of CO_2 that is naturally depleted in the atmosphere.

Key optimality conditions. The full solution of the planner's problem can be found in the Appendix. Here we report the returns of the three endogenous state variables in the model, namely a proxy for the stock of CO_2, brown capital, and green capital, respectively:

$$R^P_{s,t} = \frac{\chi_0 Y^n_t + (1-\delta_s) q^P_{s,t}}{q^P_{s,t-1}} \tag{12}$$

$$R^P_{b,t} = \left(q^P_{bt} \cdot \left[(1-\delta) + \Psi_t - \Psi'_t \cdot \frac{I_{bt}}{K_{bt-1}} \right] + \left(1 - q^P_{st} \frac{\lambda_t}{\chi_t} \right) \alpha \frac{Y^n_t}{K_{bt-1}} \right) / q^P_{bt-1} \tag{13}$$

$$R^P_{g,t} = \left(q^P_{gt} \cdot \left[(1-\delta_g) + H_t - H'_t \cdot \frac{I_{gt}}{K_{g,t-1}} \right] + q^P_{st} \kappa \frac{\tilde{G}_t}{K_{g,t-1}} \right) / q^P_{g,t-1}. \tag{14}$$

Variables denoted as q^P_{jt} refer to marginal-q, i.e., the marginal cost of an additional unit of capital of type j. The return associated to the stock of CO_2 reflects variations in the present value of future damages, $q^P_{s,t}$. The return of brown capital, $R^P_{b,t}$, differs from that in a neoclassical model because the marginal product of capital is reduced by the associated loss in value due to the additional emissions, $q^P_{st} \frac{\lambda_t}{\chi_t}$. The return of the green sector is driven by the marginal productivity of green capital, i.e., by the market value of the marginal offsets, $q^P_{st} \kappa \frac{\tilde{G}_t}{K_{g,t-1}}$.

2.2 A model with an ETS and opaque VCCs

In this section, we focus on a decentralized economy with key features that resemble the EU ETS system. We calibrate this setting to EU data and then run a counterfactual analysis of the benefits of a green coin-based system.

Green authority. The green authority observes declared net emission, $\widetilde{NE}$,

$$\widetilde{NE}_t = \lambda_t Y_t - \widetilde{G}_t,$$

where $\widetilde{G}$ refers to reported emission offsets which can differ from the true offsets, G. A penalty is applied when net emissions exceed the emission allowances, EA:

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA} \left\{ \frac{\lambda_t Y_t - EA_t}{\widetilde{G}_t} - 1 \right\} \right) \cdot Z_{t-1}.$$

In practice, this cost function mimics (i.e., it convexifies) the emission constraint

$$\lambda_t Y_t - \widetilde{G}_t - EA_t \leq 0. \tag{15}$$

The authority commits to the following rule for the supply of emission allowances to be auctioned:

$$\begin{aligned} EA_t &= \exp(a_t) \cdot Z_{g,t} > 0 \\ a_t &= (1-\rho_a)(\mu_a - \phi_a \cdot (s_{t-1} - \overline{s})) + \rho_a a_{t-1} + \sigma_a \epsilon_{a,t-j}. \end{aligned}$$

Along the balance growth path, allowances must grow with the economy, and a_t must be interpreted as a cointegration process expressed in log units. We think of this process as an AR(1) with an endogenously time-varying level,

$$\bar{a}_t := \mu_a - \phi_a \cdot (s_{t-1} - \overline{s}),$$

that is reduced when the endogenous stock of emission increases above steady state ($\phi_a > 0$). When ϕ_a is large enough, this policy implies that allowances can be kept constant or even reduced for periods when GHG emissions grows excessively.

We note three additional important points: (i) the stock of accumulated CO_2 depends on true offsets, G, not on declared offsets, $\widetilde{G}$; (ii) ϵ_a captures emission policy shocks; and (iii) we assume that the allowances supply is decided j-period ahead in order to capture the fact in the EU ETS system, the quantity of allowances to be auctioned is chosen ex-ante. In our calibration, j is one year. This exogenous policy rule can be calibrated by jointly studying emission allowances auctioned in the ETS (available at low frequency) as well as the behavior of emission prices (available at high frequency).

As discussed in what follows, we have a well-defined capital structure in the brown sector because the government offers a tax shield, τ, on corporate interest rates, $r_t^B B_{t-1}$. In addition, in the spirit of what observed in Europe, we assume that the cost of new green assets, I_{gt}, is subsidized at a rate τ_g. We assume that these payments are managed by the green authority and, for the sake of simplicity and to keep our analysis transparent, this authority has a balanced budget:

$$p_t \cdot EA_t + T_t = \tau_g I_{gt} + \tau r_t^B B_{t-1},$$

where $p_t \cdot EA_t$ is the amount collected by selling emission allowances, and T_t is a lump sum transfer from or to the household that bridges the gap between cash inflows and outflows.

Brown firm. The brown firm represents all assets and economic activities that produce positive gross emissions and faces the following problem:

$$\begin{aligned} V_{b,t} &= \max_{B_t, I_{bt}, L_{bt}, \widetilde{G}_t, EA_t} D_{b,t} + \mathbb{E}_t(M_{t+1}V_{t+1}) \\ D_{b,t} &= \underbrace{Y_t^n - W_{bt}L_{bt} - I_{bt}}_{\text{neoclassical}} \underbrace{-(1-\tau)r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\ &\quad - \underbrace{(p_t \cdot EA_t + C_t^{EA})}_{\text{cost of EA}} - \underbrace{(p_{gt}\widetilde{G}_t + C_t^C)}_{\text{cost of VCCs}} \end{aligned}$$

where Y_t^n, and K_t evolve as described in the previous section. The following cost functions,

$$\begin{aligned}
C_t^E &= \phi_0 \cdot \exp\left(\phi_1 \left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right) \cdot Z_{t-1} \quad (\text{mimics } B_t \leq \theta_t \cdot K_t), \\
\theta_t &\equiv \theta\left(\frac{\widetilde{G}_t}{\lambda_t Y_t}\right); \quad \theta'(\cdot) > 0 \\
C_t^B &= \eta Z_{t-1}\left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}}\right)^2 \\
C_t^C &= \frac{\zeta}{2}\frac{(\widetilde{G}_t - G_t)^2}{Z_{t-1}},
\end{aligned}$$

capture the cost of financial distress, debt issuance, and VCC-related reputation, respectively. As in Croce, Kung, Nguyen, and Schmid (2012), the tax shield on corporate interest makes this firm favor debt-financing (deviation from Modigliani-Miller). This aspect is important because it allows us to capture the attention of credit markets for 'net-zero' firms. Specifically, we differ from Croce, Kung, Nguyen, and Schmid (2012) by allowing the borrowing constraint tightness, θ, to be a function of declared net emissions. Hence, this firm has an incentive to buy VCCs even in the absence of an ETS, similar to what we observe in the data. As in Croce, Kung, Nguyen, and Schmid (2012), altering the mix of debt and equity comes at a cost, C^B, and hence the borrowing constraint has significant implications for investment.

In the ETS system, this firm must balance gross emissions by buying either VCCs or emission allowances to minimize the penalty cost C^{EA}. Both in the data and in the model, VCCs are on average cheaper than EU allowances, $\mathbb{E}[p_{gt}] < \mathbb{E}[p_t]$. However, VCCs carry an intrinsic cost C^C related to their opaqueness, i.e., the gap between declared and effective offsets. The firm understands that acquiring overstated offsets could lead to reputational damage and legal consequences (see, for example, the case of Delta Airlines).

The first order conditions imply that investment is distorted by the emission constraint,

$$\begin{aligned} q_{bt} &= \frac{1}{\Psi_t'} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{bt+1}}{\partial K_{bt}}\right] - \frac{\partial C_t^E}{\partial K_t} \\ \frac{\partial V_{bt}}{\partial K_{bt-1}} &= \underbrace{\frac{\partial Y_t^n}{\partial K_{bt-1}} + q_{bt}\left(1-\delta-\frac{\Psi_t' I_{bt}}{K_{bt-1}}+\Psi_t\right)}_{\text{neoclassical}} - \underbrace{\left(\frac{\partial C_t^E}{\partial K_{bt-1}}+\frac{\partial C_t^B}{\partial K_{bt-1}}\right)}_{\text{deviation from MM}} - \underbrace{\frac{\partial C_t^{EA}}{\partial K_{bt-1}}}_{\text{EA distress}}. \end{aligned}$$

In addition, the demand of emissions is related to the emission distress cost, or, equivalently, the multiplier assigned to the emission constraint,

$$p_t = \phi_1^{EA} \cdot \frac{C_t^{EA}}{G_t}.$$

Most importantly, the price of VCC is anchored to that of allowances traded on the ETS,

$$p_{gt} = \underbrace{p_t \cdot \frac{(\lambda_t Y_t - EA_t)}{\widetilde{G}_t}}_{\text{link with } p_t} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}} \underbrace{-\zeta \cdot \frac{\widetilde{G}_t - G_t}{Z_{t-1}}}_{<0,\text{ opaque}},$$

and it features two additional margins going in opposite directions. On the one hand, VCCs are more valuable than allowances because they help firms reduce their net emissions and secure better financing terms. On the other hand, because of opaqueness, VCCs are traded at a discount.

Green firm. The green sector is defined as the set of assets that delivers CO_2 offsets and sequestration. This sector determines the endogenous supply of both effective and declared offset. Specifically, the firm faces the following neoclassical problem:

$$\begin{aligned} V_{gt} &= \max_{I_{gt}, L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \\ D_{gt} &= p_{gt}\widetilde{G}_t^s - (1-\tau_{gt}) I_{gt} - W_{gt} L_{gt} \\ \widetilde{G}_t^s &= \xi_t G_t \\ \xi_t &= 1 + b_0 \exp(b_1 \epsilon_{\xi,t}), \end{aligned}$$

where $\epsilon_{\xi,t}$ is independent and identically distributed $\mathcal{N}(0,1)$, and K_{gt} and G_t are defined as in the previous section. This shock is important in order to (i) making this market opaque, and (ii) reduce the correlation between emission price, p_t, and VCC price, p_{gt}, as in the data. In equilibrium, the amount of offsets supplied by the green sector equal to the amount demanded by the brown sector, $\widetilde{G}_t^s = \widetilde{G}_t$.

We note that the positive part of the cash-flow, D_g, originates from (i) selling VCCs to the brown sector and (ii) receiving a subsidy proportional to green investment. In this formulation of the model, we think of the gap between reported and realized offsets as driven by an exogenous process, ξ_t, which is always greater than or equal to one and that captures both changes in moral hazard, ex-post lower efficiency of green assets, and changes in monitoring ability. The financing of this firm is frictionless, in order to both (i) capture the 'benevolence' that capital markets are giving to green projects and (ii) isolate the role of financing frictions only on brown firms.

At the optimum, we have

$$\begin{aligned} q_{gt} &:= \frac{1-\tau_{gt}}{H_t'} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \\ \frac{\partial V_{gt}}{\partial K_{g,t-1}} &= p_{g,t}\frac{\partial \widetilde{G}_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t'\cdot(I_{gt}/K_{g,t-1})\}, \end{aligned}$$

implying that green investment is driven by the present value of the future offset flows. This flow is crucially driven by the price of VCCs which, in turn, is anchored to the price of allowances on the ETS.

Aggregate resource constraint and consumer budget constraint. Let V_{jt}^{ex} denote the ex-dividend price of asset j at time t, i.e., $V_{jt}^{ex} = V_{jt} - D_{jt}$. The household budget constraint is defined as follows:

$$C_t + V_{bt}^{ex} + V_{gt}^{ex} = V_{bt} + V_{gt} + W_{bt} + W_{g,t} - T_t$$

Since offsets are sold to the brown firm, they represent intermediate goods that do not show up in GDP. Therefore, we have

$$Y_t^n = C_t + I_{bt} + I_{gt} + C_t^{EA} + C_t^B + C_t^E + C_t^C.$$

2.3 A Model with Green Coins

The economy described in the previous section features many sources of inefficiencies, and it is far away from the first-best. In what follows, we consider a setting in which there is scope for a central authority to improve welfare. Specifically, we think of an authority called Green Coins Central Bank (GCCB) that runs and monitors the only authorized system in which offsets are recorded, verified, and monetized or, equivalently, tokenized. The GCCB can alter the price of the green coins with open market operations to bring the economy closer to the first best.

In this setting, offsets are properly measured, but allocations can still be inefficient due to the presence of financial frictions. We will show that the GCCB can improve welfare by responding to exogenous shocks in ways that make investment activities in both the brown and green sector to be more efficient from a social perspective.

Unmodeled frictions. In the context of our model, all we need is to assume that the GCCB has a verification technology and an exchange in which to trade offset units. The model is silent on whether the verification of the transaction should be centralized or decentralized. In what follows, we assume that the system runs on a decentralized blockchain and use blockchain-related terminology. We believe that this technology is superior in at least three dimensions. First, it is very scalable and can be adopted by large, medium, and small businesses. Equivalently, it features smaller adoption fixed costs. Second, in an economy with multiple agents, a decentralized verification of the offsets transactions minimizes the incentives for misreporting. Third, since offsets and emissions may be localized in very different countries, a common blockchain can (i) homogenize offsets accounting and (ii) minimize country-level incentives to overstate offsets and under-report emissions.

The green blockchain. We assume that the GCCB runs a blockchain that mines new coins every period as soon as an offset is verified. Coins are fully retired at the end of the period. As a result, the total supply of green coins in each period equals the total mass of true offsets in every period and every state of the world, G_t. The per-period cost of running the blockchain is $\omega_g G_t$, i.e., it is proportional to all transactions which are verified by the platform. In our model, the green firm is the only miner of green coins, and it has an incentive to sell them to the brown firms who can claim 1 unit of offsets for each coin. The brown firm buys green coins at the spot price p_g. In this setting, all emissions and offsets are verified over the platform, and hence there is no VCC.

The role of the GCCB. In each period, the green authority conducts open market operations in order to buy a share $S_{g,t}$ of the total mass of available green coins, G_t. Thus the total supply of offsets accessible to the brown sector is

$$G_t^P := G_t \cdot (1 - S_{g,t}),$$

and the private sector is now subject to the following constraint:

$$NE_t = \underbrace{\lambda_t Y_t}_{\text{gross emissions}} - \underbrace{G_t^P}_{\text{private offsets}} \leq \underbrace{\bar{A} \cdot Z_{g,t-1}}_{\text{emissions allowed}} . \tag{16}$$

By altering the supply of green coins available to the public, the GCCB can manipulate in real time the price of green coins, i.e., the only assets that count for emission regulation. This formulation resembles what central banks do with bonds in order to alter yields. Note that thanks to the monitoring implemented by the blockchain, there is no longer a distinction between true offsets, G, and declared offset ($G_t = \tilde{G}_t \quad \forall t$).

The authority is assumed to follow this Taylor-style green rule,

$$S_{gt} = \frac{\exp(sg_t)}{1+\exp(sg_t)} \in (0,\, 1),$$

where

$$sg_t = A_0 + [A_s\, A_k\, A_{lr}] \cdot \begin{bmatrix} \hat{s}_{t-1} \\ \widehat{\frac{K_{gt-1}}{K_{bt-1}}} \\ x_{t-1} \end{bmatrix} + [A_z\, A_\lambda\, A_x] \cdot \begin{bmatrix} \epsilon_{z,t} \\ \epsilon_{\lambda,t} \\ \epsilon_{x,t} \end{bmatrix}, \tag{17}$$

so that the holdings of green coins is both history dependent (it depends of the demeaned state variables, $\hat{s}_{t-1}$, $\widehat{\frac{K_{gt-1}}{K_{t-1}}}$, and x_{t-1}) and sensitive to all contemporaneous shocks in the economy ($\epsilon_{z,t}$, $\epsilon_{\lambda,t}$, and $\epsilon_{x,t}$). The coefficients of this rule cannot be estimated in the data since this is a counterfactual setting. In our analysis, we will explore the welfare implications of allowing the GCCB to respond to economic states by changing these parameters.

The authority net payout to the household is:

$$\Pi_t^{GB} = -(p_{g,t} \cdot G_t \cdot S_{g,t} + \omega_g \cdot G_t + \tau_g I_{gt} + \tau r_t^B B_{t-1}),$$

which embodies the resources spent to buy new coins and running the blockchain.

Brown Firm. The new problem faced by the brown firm is:

$$\begin{aligned} V_{b,t}^{GC} &= \max_{B_t, I_{bt}, L_{bt}, G_t} D_{b,t}^{GC} + \mathbb{E}_t(M_{t+1}V_{t+1}) \\ D_{b,t}^{GC} &= \underbrace{Y_t^n - W_{bt}L_{bt} - I_{bt}}_{\text{neoclassical}} \underbrace{-(1-\tau)r_t^B B_{t-1} + \Delta B_t - C_t^E - C_t^B}_{\text{deviation from MM}} \\ &\quad - \underbrace{C_t^{EA}}_{\text{cost of net emissions}} - \underbrace{p_{gt}G^P{}_t,}_{\text{cost of coins}} \end{aligned}$$

in which there are no longer allowances traded on the ETS and VCCs, only green coins. The variables Y_t^n, K_{bt}, C_t^E, C_t^B, are defined as in the setting with the ETS system. The C_t^{EA} cost function is formulated to reflect the emission constraint in equation (16) as opposed to equation (15):

$$C_t^{EA} = \phi_0^{EA} \cdot \exp\left(\phi_1^{EA}\left\{\frac{(\lambda_t Y_t - \bar{A}Z_{g,t-1})}{G_t^P} - 1\right\}\right) \cdot Z_{t-1}.$$

At the optimum, the price of green coins is

$$p_{gt} = \underbrace{\phi_1^{EA} \cdot \frac{C_t^{EA}}{G_t^P} \cdot \frac{(\lambda_t Y_t - \bar{A}Z_{g,t-1})}{G_t^P}}_{\text{savings on the cost of NE>0}} + \underbrace{\phi_1 \cdot \theta_t' \cdot \frac{C_t^E}{B_t} \cdot \frac{K_{bt}}{\lambda_t Y_t}}_{\text{savings on borrowing}}.$$

Importantly, this price still reflects both the benefits related to cheaper financing of low net-emission firms and the tightness of the emission constraint. In contrast to the previous setting, now the GCCB can directly control the emission constraint tightness by altering its holding of green coins.

Green firm. The green sector determines the endogenous supply of green coins. Specifically, the firm faces the following neoclassical problem:

$$\begin{aligned} V_{gt}^{GC} &= \max_{I_{gt},L_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1}V_{g,t+1}) \\ D_{gt} &= p_{gt}G_t - (1-\tau_{gt})I_{gt} - W_{gt}L_{gt}, \end{aligned}$$

where all variables are defined as before and there is no longer any wedge between declared and actual offsets.

Aggregate resource constraint and consumer budget constraint. In this setting, The household budget constraint is defined as follows:

$$C_t + V_{bt}^{ex,GC} + V_{gt}^{ex,GC} = V_{bt}^{GC} + V_{gt}^{GC} + W_{bt} + W_{g,t} + \Pi_t.$$

Since offsets are sold to the brown firm, they represent intermediate goods that do not show up in GDP. Therefore, we have

$$Y_t^n = C_t + I_{bt} + I_{gt} + C_t^{EA} + C_t^B + C_t^E + \omega_g G_t.$$

3 Data and Calibration

In what follows, we describe the data that we use in order to calibrate our main specification to European data. We then illustrate our calibration and solution methods.

3.1 Data

We focus on the EA19, i.e., the 19 countries in the Euro Area as of 2015, in order to maximize our sample length across time series. All macroeconomic series on national aggregates and price indices are from Eurostat.

Data on green investments and prices for futures related to CO_2 offsets are from Bloomberg. Specifically, with regard to the CME commodity market exchange, we consider daily prices for "XGB1 Comdty: CBL CORE GLOBAL EMISSIONS OFFSET (C-GEO) FUTURES - TR", "LGO1 Comdty: Generic 1st CBL Nature-Based Global Emissions Offset Futures", and "EEXX04EA Index: EEXX European Emission Allowances EUA Spot (Phase 4)" over the sample 2021-2023. In addition, we recover green investments by country in billions of Euros at the yearly frequency over the 2004-2023 sample.

Haver Analytics enables us to obtain daily observations over the sample 2005-2024 for futures and emission allowances "ICE ECX EUA Futures: 1st Position: Settlement Price (Euro/Metric Tonne)", "CME Reg Sess:Carbon Offset Futures: 1st Position: Settlement Price (US/Offset)" and "NYMEX RSes:CBL Nat-Based Glob Emiss Offset".

Emissions as percent of GDP by country at the annual frequency for the 2010-2020 sample are from The United Nations Economic Commission for Europe (UNECE). We complement

Table 1: Calibration

Parameter	Value	Parameter	Value
Preferences		***Climate***	
Discount factor (β)	0.97	GHG depletion rate % (δ_s)	0.55
Risk aversion (γ)	10.00	Mean of log emission rate (μ_λ)	-1.86
IES (ψ)	1.20	Persistent of log emission rate (ρ_λ)	0.50
Damage function parameters % (χ_0)	0.16	Log emission rate volatility (σ_λ)	0.02
GCCB		***VCCs***	
Cost of running blockchain % (ω_g)	0.39	Reporting gap avg. % (b_0)	2.20
Average emissions allowance ($\bar{A} = e^{\mu_a}$)	0.15	Reporting gap volatility (b_1)	0.50
		Reporting costs intensity (ζ)	15.44
Technology (Brown sector)		***Technology (Green sector)***	
Capital share (α)	0.35	Capital share (κ)	0.60
Capital depreciation rate % (δ)	7.60	Capital depreciation rate % (δ_g)	7.60
Elasticity of investment adj. costs (ω)	1.50	Elasticity of investment adj. costs (ν)	15.00
Intensity of debt adjustment costs (η)	0.40	Intensity of emission violation costs (ϕ_1^{EA})	20.00
Debt-to-book avg (θ_0)	0.50	Emission violation costs parameter % (ϕ_0^{EA})	0.05
		Green-brown differences in levels of TFP ($\overline{zc}$)	-0.11
Productivity		***ETS Authority***	
Average productivity growth (μ_z)	0.01	Brown-sector debt interest tax deductible (τ)	0.20
Short-run productivity volatility (σ_z)	0.05	Green-sector investment subsidy (τ_g)	0.30
Long-run productivity persistence (ρ_x)	0.94	Mean emission allowances (μ_a)	-1.92
Long-run productivity volatility % (σ_x)	0.50	Persistent of emission allowances (ρ_a)	0.27
Long-run productivity exposure		Allowance shock volatility (σ_a)	0.12
to GHG emissions (ϕ_s, $\times$ 1000)	0.20	Allowance sensitivity to GHG emissions % (ϕ_a)	8.71
Cointegration parameter (ϕ_g)	0.20	Intensity of distress costs (ϕ_1)	20.00

Notes: This table reports our benchmark annual calibration. See section 3.2 for a detailed discussion.

these data with those from the Clean Investment Monitor. This dataset refers mostly to green investments done in the US energy and manufacturing sectors and can be interpreted as providing a lower bound on investment over the sample 2018:Q1-2024:Q1.

Verified carbon credits data are obtained mostly from the Goldstandard Registry of projects that offset emissions. We complement this source with the data on sustainable projects reported by both the International Renewable Energy Agency (IRENA) and the International Energy Agency (IEA).

3.2 Calibration and Solution Method

We report our benchmark calibration for the model with the ETS system in table 1. The main moments that we target are reported in table 2. The AE19 economy is not too dissimilar

Table 2: Simulated Moments

	Data		Model		
	Point Est.	S.Err.	ETS	Green Coins	Planner
Standard Moments					
$\mathbb{E}(\Delta c)$ (%)	0.96	0.40	1.00	1.00	1.00
$\sigma(\Delta c)$ (%)	2.22	0.57	2.70	2.70	3.10
$ACF1(\Delta c)$	0.03	0.23	0.04	0.02	-0.01
$\sigma(\Delta c)/\sigma(\Delta gdp)$	0.73	0.12	0.83	0.83	0.95
$\sigma(\Delta i)/\sigma(\Delta gdp)$	1.98	0.28	1.58	1.49	1.14
$\mathbb{E}(C)/E(GDP)$	0.72	0.43%	0.74	0.74	0.73
$\mathbb{E}(r_f)$ (%)	3.39	2.09	2.52	2.56	2.41
$\mathbb{E}(r^B)$ (%)	6.27	6.52	6.67	6.79	3.73
$\mathbb{E}(r^B - r^G)$ (%)	3.34	1.98	4.29	4.59	1.45
Emissions, ETS, and VCC prices					
$\mathbb{E}(p)/\mathbb{E}(p_g)$	4.15	1.87	1.85		
$\mathbb{E}(\log(\lambda))$	-1.98	0.07	-1.86	-1.86	-1.86
$\sigma(\log(\lambda))$	0.02	0.22%	0.02	0.02	0.02
$ACF1(\log(\lambda))$	0.29	0.17	0.40	0.40	0.40
$\mathbb{E}(a)$	-1.92	0.12	-1.92		
$\sigma(a)$	0.12	0.02	0.12		
$ACF1(a)$	0.27	0.20	0.20		

Notes: Empirical moments are computed using annual data. All data sources are discussed in 3.1. Standard errors are calculated using HAC-adjusted standard errors. The entries for the model are obtained by averaging the results across simulated small samples. Our baseline calibration is detailed in table 1. ACF(1) stands for autocorrelation function with a lag equal to 1.

from the US one in terms of basic macroeconomic statistics. As a result, our calibration is similar to the one in prior work (see, among others, Croce 2014 and Croce, Kung, Nguyen, and Schmid 2012). Damage specification and parameters are in the spirit of those in Shapiro and Metcalf (2023).

The greenium is computed by sorting European public firms according to their direct emission-intensity reported in Trucost. We use the returns of the top (bottom) quintile to measure the return of brown (green) firms in our model. Returns are equally weighted. Looking at equally-weighted returns of emission intensity-sorted firms mitigates bias concerns related to size.

The ratio $\mathbb{E}(p)/\mathbb{E}(p_g)$ considers the average cost of emissions on the European ETS versus the average cost of emissions on the private VCC market. As mentioned in the previous section, data on emissions-per-real-GDP, λ, are from the UNECE. Data on allocated allowances traded on the ETS are from the European ETS system.

We solve our models using the perturbation method. Specifically, we use `Dynare++` to implement a second-order log approximation. We find the performance of our model very satisfactory, given that it reproduces several dimensions of both the macro and the emission data.

4 Main Results

In this section, we share results relevant to show how our model could be used for policy analysis.

4.1 Optimal GCCB Policy Rule.

Given the encouraging results obtained from calibrating the model with the ETS setting, we now proceed with the calibration of our counterfactual economy with green coins. In order to do so, we need to calibrate the parameters in equation (17). Since there is no data counterpart, we collect all of these parameters in a vector θ and choose them in order to solve the following problem:

$$\theta^* := \arg\min_{\theta} \max\{\mathbb{E}\left[(R^P_{bt} - R^{GC}_{bt}(\theta))^2\right], \ \mathbb{E}\left[(R^P_{gt} - R^{GC}_{gt}(\theta))^2\right]\}, \tag{18}$$

where R^P_{bt} and R^P_{gt} are defined in equations (13)–(14) and refer to the return of brown and green capital chosen by planner, respectively. The returns R^{GC}_{bt} and R^{GC}_{gt}, defined in the Appendix, equation (A36) and (A44), respectively, are obtained by solving the setting with green coins given a specific value of θ.

In practice, we simulate both the planner's problem and the green coin economy using the same set of common exogenous shocks and minimize the maximum average discrepancy in the returns of brown and green capital across states and histories. By doing so, we

Table 3: Optimal GCCB Policy Parameters

	A_0	A_s	A_k	A_{lr}	A_λ	A_x	A_z
θ^*	-0.157	1685	-1068	-471	-4.25	-0.58	-2.78
$\frac{\partial u}{\partial(\cdot)}(\times 100)$	78.68	-0.07%	-0.11%	-0.011%	0.08%	-0.005‰	1.38%

Notes: This table reports the values of the parameters determined as in equation (18). In the second row, we report the partial marginal variation of the welfare function with respect to each parameter.

identify a green coin policy that mimics as much as possible the dynamics of the returns and capital stocks that we would observe at the first-best. As shown our figures (C1)–(C2) in the appendix, the biggest average discrepancy is driven by the returns of green capital. As a result, the optimal parameters are mainly driven by the green sector. All parameters belong to the interior, with the exception of A_0. In our model, we require total GHGs concentration to be non-negative, and thus the number of green coins the authority hold cannot be too large. That explains the corner solution with respect to A_0.

Policy parameters and implications. We report our parameter values in first row of table 3. First of all, the value of A_0 implies that it is optimal for the GCCB to hold on average 46% of the total coins, i.e., the institution must commit to a quite 'hawkish' target. The GCCB increases this share with the stock of CO_2 ($A_s > 0$), but it also accommodates long-run growth ($A_{lr} < 0$) in the overall economy. In addition, the GCCB reduces its holdings of coins when the green sector has become particularly large compared to the brown one ($A_k < 0$).

The rightmost three coefficients refer to the contemporaneous response to the three unexpected shocks in the economy. In all cases, the GCCB responds by increasing the supply of green coins available to the private sector, i.e, by reducing its share of coins. We note that this behavior corresponds to implement an 'emission-smoothing policy'.

Specifically, the emission constraint produces costs proportional to output, and hence it has effects similar to those of distortionary taxes. Under the optimal policy, it is optimal to accommodate expansionary productivity shocks by leaving more green coins to the private sector and letting their price decline. Note that accommodating these shocks in the short-run cause an increase in the future stock of CO_2, s. In addition, to the extent that this shock diffuse slowly to the green sector, green capital decreases relative to brown capital. These two channels reinforce each other, and they imply that the short-run accommodation slowly turns into a more stringent emission constraint for the long-run.

Similarly, if the economy is affected by an adverse shock to the emission-to-GDP intensity, $\epsilon_\lambda > 0$, the GCCB responds by partially offset this shock by leaving more coins to the private sector. As the stock of CO_2 increases, the GCCB increases its purchases of coins in the long-run making them more expensive for the private sector.

Welfare analysis. It is important to assess the convenience of the token-based system compared to the ETS setting. First of all, we must point out that in the setting with the ETS, the welfare costs are equal to 6.93% of time-0 first-best consumption. Our green coin setting overcomes most of this loss as it features a welfare loss of just 0.05% of first-best time-0 consumption. We compare simulated moments across these settings in table 2. In what follows, we 'inspect the mechanism' in order to detail where these welfare gains come from.

The second row of table 3 shows the sensitivity of our welfare function in the settings with green coins with respect to each parameter evaluated at θ^*. Looking at the magnitudes of these derivatives, the most important parameters that we discuss are A_0 and A_z. In figure 3, we plot average moments as we vary these two parameters. Specifically, we focus on how these parameters affect the relative size of the green and brown sector, the average damage, and the risk premia for the green and brown sectors.

In each panel of figure 3 we plot two lines. The dotted (solid) lines refer to difference between the moment obtained in our green coin setting and the corresponding value at the first-best (under the ETS setting). Hence when the red (blue) line gets close to zero, it means that the green coin-based setting mimics the planner (ETS setting) stochastic steady

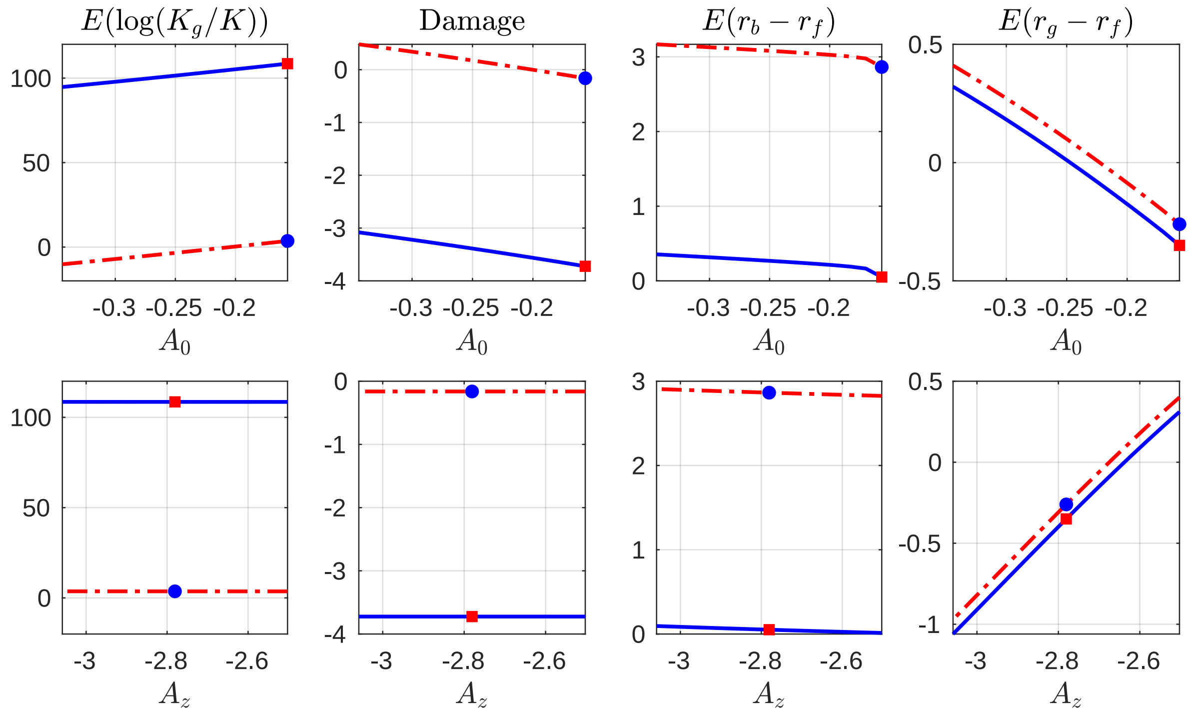

Fig. 3: Policy Parameters and Stochastic Steady State Values

Notes: In each panel, the blue (red) line shows the difference between the reported moment computed under the green coin setting and the ETS (first-best) setting. All variables are multiplied by 100.

state.

A_0 is a key parameter that allows the green coin setting to mimic the same relative size of the green sector obtained at the first-best. As a result, the implied average damage in this economy is very similar to the first-best level. The ETS setting, instead, features a much smaller green sector and a bigger damage. Interestingly, by committing ex-ante to buy a higher average share of green coins, the GCCB sustains the green sector and reduces its cost of capital while keeping the cost of equity for the brown sector almost constant. At the optimum, the cost of equity in the brown sector is very similar to the level obtained under the ETS system, whereas the cost of capital of the green sector is reduced even below its first-best level. We note that our setting with green coins is still affected by financing frictions, and hence it is not supposed to perfectly mimic the first-best. Given the implicit subsidy to brown capital through the tax shield on debt, the GCCB subsidizes the green sector through a policy that makes the cashflow associated to the offsets safer. By no arbitrage, a lower risk premium stimulates green investment. For a complete comparison of the ETS and the green coin-based economy simulated moments, see table 2.

Finally, we note that the riskiness of both the green and the brown sector is sensitive to A_z, i.e., the contemporaneous response of the GCCB to short-run growth shocks. If this parameter is not negative enough, i.e., if the GCCB does not accommodate positive shocks, the risk premium increases. Simultaneously, in the brown sector a less accommodating policy results in cashflows that respond less to long-run news. This is because of hedging effect coming from the cost of offsetting emissions. As a result, as A_z becomes less negative, the risk premium in the brown sector declines. These effects on the cost of capital ultimately affect the relative size of the green and brown sectors. Specifically, a less accommodating policy response to an emission intensity shock reduces the relative share of green capital in the economy. The latter effect is not easily visible in our panel because it is second-order compared to the difference in our steady state moments across the ETS and the first-best settings.

4.2 Dynamic Responses

Analysis the dynamic responses to shocks across the three settings help us further in understanding in which dimensions that green coin setting dominates the ETS. In what follows, we discuss the responses depicted in figure 4.

With respect to a positive short-run productivity shocks, consumption, brown investment, and output behave as in a standard neoclassical model. Under the first best, there is a reallocation of investment away from the green sector in favor of the brown sector. This is because the productivity gain affects immediately the brown sector, but it propagates slowly over time in the green sector. In the ETS economy, in contrast, the opposite happens. The emission allowances do not adjust promptly to this shock and hence there is a strong demand for VCCs which is followed by more investment in green assets. Also the green coin

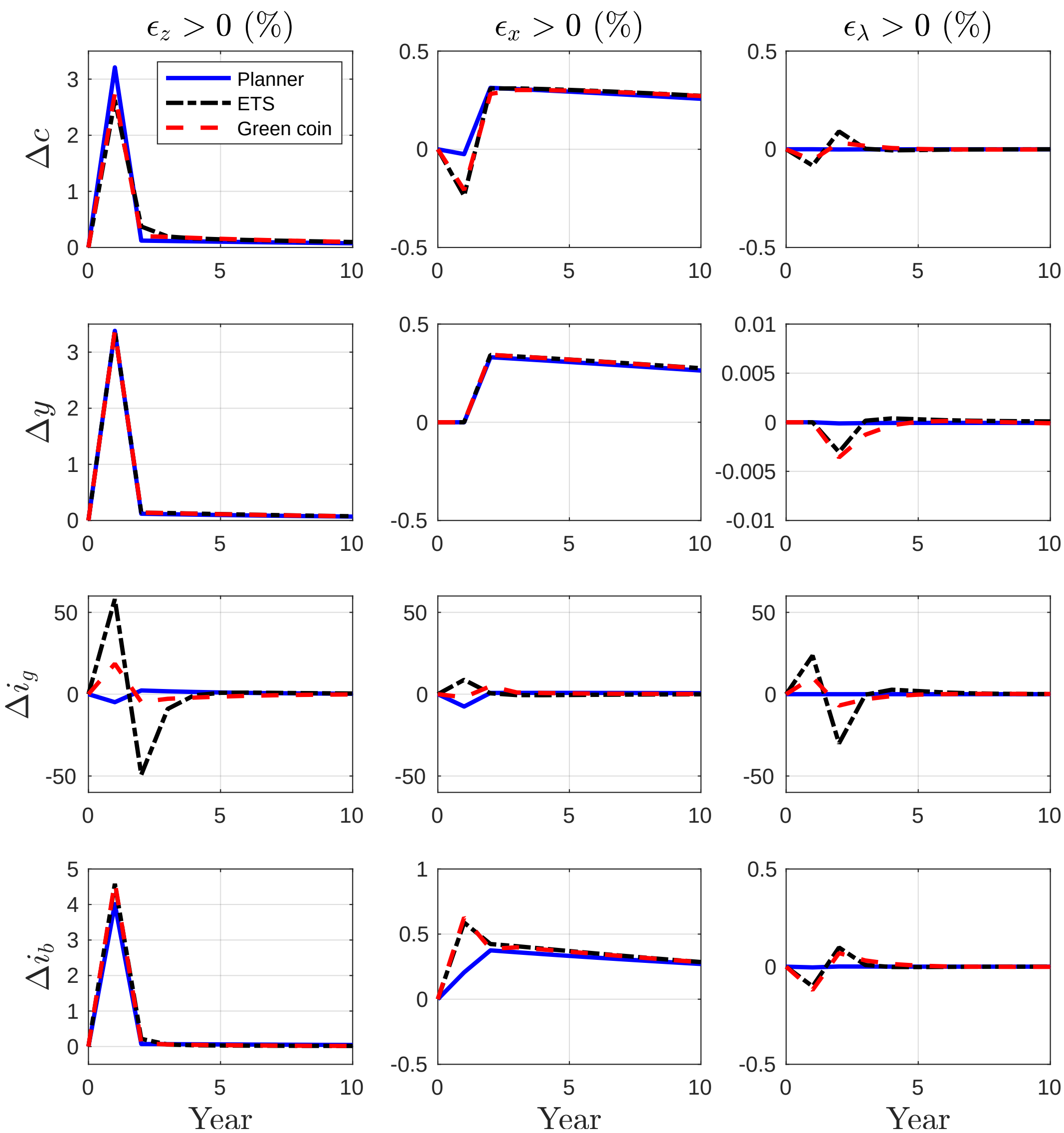

Fig. 4: Impulse Responses: Quantities

Notes: This figure depicts responses to a one-standard deviation shock. All responses are multiplied by 100. The benchmark calibration is detailed in table 1.

setting features an increase in green investment, but this increase is moderate and hence this response is closer to that measured at the first-best.

When we turn our attention to long-run productivity news, the green investment in the green coin setting mimics very closely the planner's solution. Both the ETS and the green coin settings feature excessive increase in investment and this is due to the additional distortion coming from leverage (Croce, Kung, Nguyen, and Schmid, 2012).

In the ETS system, a positive emission shock, λ, promote a reallocation of investment resources. A higher λ prompts a higher demand of VCCs, and hence it increases green investment. Simultaneously, by resource constraint, brown investment must decline. Because of convex adjustment costs, these adjustments also imply that green assets appreciate, and thus they provide an hedge against emission shocks. Brown stocks, instead, depreciate in a high marginal utility state, and hence they carry an additional risk-premium. These difference in exposure generates a greenium. Interestingly, at the first-best, the response of macroeconomic quantities with respect to this shock go in the same direction, but they are two orders of magnitude smaller. Our green coin setting promotes a reallocation of investment resources that is in between that observed in the ETS setting and at the first-best. Hence, this setting mitigates distortions also with respect to changes in the emission-to-output ratio.

When we turn our attention to the cost of carbon across our three settings. We report our model implied values in table 4. Note that these values refer to marginal prices rescaled by productivity. In order to better interpret these results, we also report the implied equilibrium discount rate for future damages. The marginal cost of CO_2 declines as we move from the Planner's setting toward the ETS system. At first sight, one may find this result surprising as our welfare results go exactly in the opposite direction. In reality, this result is fully driven by the discount rate. The cost of CO_2 is a present value, and it declines with higher discount rate. In the ETS system, the damage is more procyclical, and thus it carries a higher risk premium. The marginal cost of carbon does not reflect welfare because it is silent on the quantity of emissions. In our model, emissions under the ETS system are 20 times higher than at the first-best and hence the implied damage produces significant welfare losses.

Table 4: Emission Price and Discount Rate

Planner		Green Coins		ETS	
q_s	r_s	q_s	r_s	q_s	r_s
0.073	4.28	0.071	4.33	0.067	4.44

Notes: This table reports the implied cost of emissions scaled by productivity and its discount rate across our three settings.

5 Conclusions

We propose a new model to examine emissions regulation in general equilibrium. We study an inefficient decentralized economy with an ETS, opaque VCCs, and credit markets with a bias for 'net-zero' firms. We show that this setting is far away from the first-best. We then analyze a counterfactual setting in which emission offsets are tokenized, and tokens available to the private sectors are managed by a Green Coin Central Bank (GCCB) that controls the cost of emissions with state-contingent open market operations. We show that this setting can produce substantial welfare gains.

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A Planner's problem

The Lagrangian for the planner's problem:

$$\mathcal{L} = \left[(1-\beta)\tilde{C}_t^{1-\frac{1}{\psi}} + \beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}}$$

$$+ \{(1-\delta)K_{t-1} + \Psi(I_t/K_{t-1})K_{t-1} - K_t\}Q_t$$

$$+ \{(1-\delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1} - K_{gt}\}Q_{g,t}$$

$$+ \{\chi(s_t)(Z_tL_t)^{1-\alpha}K_{t-1}^{\alpha} - (\tilde{C}_t + I_{gt} + I_t)\}\Omega_t$$

$$+ \left\{s_t - \left((1-\delta_s)s_{t-1} + \{\lambda_t(Z_tL_t)^{1-\alpha}K_{t-1}^{\alpha} - (Z_{gt}L_{gt})^{1-\kappa}K_{g,t-1}^{\kappa}\}/Z_{t-1}\right)\right\} Q_{st},$$

where Q_t, $Q_{g,t}$, Ω_t, and $Q_{s,t}$ are multipliers. The first order conditions are

$$0 = U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\,\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial K_t}\right) - Q_t \tag{A1}$$

$$q_t := \frac{Q_t}{\Omega_t} = 1/\Psi'(I_t/K_{t-1}) \tag{A2}$$

$$0 = U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\,\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial K_{gt}}\right) - Q_{gt} \tag{A3}$$

$$q_{gt} := \frac{Q_{g,t}}{\Omega_t} = 1/H'(I_{gt}/K_{g,t-1}) \tag{A4}$$

$$0 = (1-\beta)U_t^{1/\psi}\tilde{C}_t^{-\frac{1}{\psi}} - \Omega_t \tag{A5}$$

$$0 = -\chi_0\cdot\chi_t\cdot Y_t\Omega_t + U_t^{1/\psi}\beta(\mathbb{E}_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}-1}\,\mathbb{E}_t\left(U_{t+1}^{-\gamma}\frac{\partial U_{t+1}}{\partial s_t}\right) + Q_{st}. \tag{A6}$$

Plug (A5) into (A1), (A3), and (A6), we obtain

$$q_t = \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\frac{\partial U_{t+1}}{\partial K_t}\right) \tag{A7}$$

$$q_{gt} = \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\frac{\partial U_{t+1}}{\partial K_{gt}}\right) \tag{A8}$$

$$\hat{q}_{st} := \frac{Q_{st}}{\Omega_t} = \chi_0\cdot\chi_t\cdot Y_t + \mathbb{E}_t\left(M_{t+1}\frac{1}{\Omega_{t+1}}\cdot\left(-\frac{\partial U_{t+1}}{\partial s_t}\right)\right) \tag{A9}$$

$$\hat{p}_{st} := \mathbb{E}_t\left(M_{t+1}\hat{q}_{s,t+1}\right) \tag{A10}$$

$$\hat{q}_{st} = \chi_0 \cdot \chi_t \cdot Y_t + (1-\delta_s)\hat{p}_{st}. \tag{A11}$$

The envelop conditions read

$$\frac{\partial U_t}{\partial K_{t-1}} = Q_t \cdot \left[(1-\delta) + \Psi_t - \Psi_t' \cdot \frac{I_t}{K_{t-1}}\right] + \left(\Omega_t \chi_t - \frac{Q_{st}}{Z_{t-1}}\lambda_t\right)\alpha\frac{Y_t}{K_{t-1}} \tag{A12}$$

$$\frac{\partial U_t}{\partial K_{g,t-1}} = Q_{gt} \cdot \left[(1-\delta_g) + H_t - H_t' \cdot \frac{I_{gt}}{K_{g,t-1}}\right] + \frac{Q_{st}}{Z_{t-1}}\kappa\frac{\tilde{G}_t}{K_{g,t-1}} \tag{A13}$$

$$\frac{\partial U_t}{\partial s_{t-1}} = -(1-\delta_s)Q_{st}. \tag{A14}$$

After defining $q_{st} = \hat{q}_{st}/Z_{t-1}$ and $p_{st} = \hat{p}_{st}/Z_{t-1}$, we obtain the returns as reported in the paper

$$R^P_{s,t} = \frac{Z_{t-1}}{Z_{t-2}}\frac{q_{st}}{p_{s,t-1}} \tag{A15}$$

$$R^P_{I,t} = \left(q_t \cdot \left[(1-\delta) + \Psi_t - \Psi_t' \cdot \frac{I_t}{K_{t-1}}\right] + (\chi_t - q_{st}\lambda_t)\,\alpha\frac{Y_t}{K_{t-1}}\right)/q_{t-1} \tag{A16}$$

$$R^P_{G,t} = \left(q_{gt} \cdot \left[(1-\delta_g) + H_t - H_t' \cdot \frac{I_{gt}}{K_{g,t-1}}\right] + q_{st}\kappa\frac{\tilde{G}_t}{K_{g,t-1}}\right)/q_{g,t-1}, \tag{A17}$$

and the associated Euler equations:

$$1 = \mathbb{E}_t\left[M_{t+1}R^P_{s,t+1}\right] \tag{A18}$$

$$1 = \mathbb{E}_t\left[M_{t+1}R^P_{I,t+1}\right] \tag{A19}$$

$$1 = \mathbb{E}_t\left[M_{t+1}R^P_{G,t+1}\right] \tag{A20}$$

B ETS with opague VCCs

B.1 Brown firm

The brown firm solves

$$V_t = \max_{K_t, B_t, I_t, G_t, EA_t} D_t + \mathbb{E}_t(M_{t+1}V_{t+1}) \tag{A21}$$

subject to

$$\begin{aligned} D_t &= \chi(s_t) \cdot Y_t - W_t L_t - (1-\tau) r_t^B B_{t-1} + \Delta B_t - I_t - C_t^E - C_t^B \qquad \text{(A22)} \\ &\quad - p_t \cdot EA_t - p_{gt} G_t - C_t^C - C_t^{EA} \\ K_t &= (1-\delta) K_{t-1} + \Psi(I_t / K_{t-1}) K_{t-1} \qquad \text{(A23)} \end{aligned}$$

Optimality with respect to debt. The first order condition with respect to corporate debt and the Envelope theorem jointly imply:

$$\begin{aligned} 1 &= \frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} - \mathbb{E}_t \left[M_{t+1} \frac{\partial V_{t+1}}{\partial B_t} \right] \\ \frac{\partial V_t}{\partial B_{t-1}} &= -(1 + (1-\tau) r_{t-1}^B). \end{aligned}$$

Taking into account the fact that at the equilibrium r^B is equal to the risk-free rate:

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \mathbb{E}_t \left[M_{t+1} \tau \right] r_{f,t},$$

which becomes

$$\frac{\partial C_t^B}{\partial B_t} + \frac{\partial C_t^E}{\partial B_t} = \frac{\tau \cdot r_{f,t}}{1 + r_{f,t}},$$

The left-hand side refers to the total marginal cost of issuing an extra unit of debt. The right-hand side measures the value of the corporate interest tax advantage.

Optimality with respect to EA. The demand of emission allowances is related to the marginal environmental distress cost:

$$p_t = -\frac{\partial C_t^{EA}}{\partial EA_t} = \phi_1^{EA} \frac{C_t^{EA}}{G_t}$$

Optimality with respect to declared green credits. The first order condition with respect to G_t imply the following:

$$p_{gt} = \underbrace{p_t \cdot \frac{\lambda_t Y_t - EA_t}{G_t}}_{\text{savings on EA}} \quad \underbrace{-\frac{\partial C_t^E}{\partial G_t}}_{>0\text{, savings on borrowing}} \quad \underbrace{-\zeta \cdot \frac{G_t - \tilde{G}_t}{Z_{t-1}}}_{<0}$$

Optimality with respect to capital. Envelope and first order conditions with respect to investment and capital imply the following:

$$\begin{aligned} q_t &= \frac{1}{\Psi_t'} = \mathbb{E}_t \left[M_{t+1} \frac{\partial V_{t+1}}{\partial K_t} \right] - \frac{\partial C_t^E}{\partial K_t} \\ \frac{\partial V_t}{\partial K_{t-1}} &= \chi(s_t) \frac{\partial Y_t}{\partial K_{t-1}} - \frac{\partial C_t^{EA}}{\partial K_{t-1}} - \frac{\partial C_t^E}{\partial K_{t-1}} - \frac{\partial C_t^B}{\partial K_{t-1}} + q_t \left(1 - \delta - \frac{\Psi_t' I_{bt}}{K_{t-1}} + \Psi_t \right). \end{aligned}$$

B.2 Green firm

$$V_{gt} = \max_{K_{gt}, I_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1} V_{g,t+1}) \tag{A24}$$

$$D_{gt} = p_{gt} G_t^s - (1 - \tau_{gt}) I_{gt} - W_{gt} L_{gt} \tag{A25}$$

$$K_{gt} = (1 - \delta_g) K_{g,t-1} + H(I_{gt}/K_{g,t-1}) K_{g,t-1}, \tag{A26}$$

where $G_t^s = \xi_t \tilde{G}_t$; $\xi_t = 1 + b_0 \exp(b_1 \epsilon_{\xi,t})$; and $\tilde{G}_t = (Z_{g,t} L_{gt})^{1-\kappa} K_{g,t-1}^{\kappa}$.

Investment are determined by

$$q_{gt} = (1-\tau_{gt})/H_t' = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \tag{A27}$$

$$\frac{\partial V_{gt}}{\partial K_{g,t-1}} = p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1-\delta_g + H_t - H_t'\cdot(I_{gt}/K_{g,t-1})\} \tag{A28}$$

C Green Coin Setting

C.1 Brown firm

The brown sector faces the following problem

$$V_t = \max_{K_t, B_t, I_t, G_{gt}^{PD}} D_t + \mathbb{E}_t(M_{t+1}V_{t+1}) \tag{A29}$$

$$D_t = \chi(s_t)\cdot Y_t - W_tL_t - (1-\tau)r_t^B B_{t-1} + \Delta B_t - I_t - C_t^E - C_t^B \tag{A30}$$

$$- p_{gt}G_{gt}^{PD} - C_t^{EA}$$

$$K_t = (1-\delta)K_{t-1} + \Psi(I_t/K_{t-1})K_{t-1} \tag{A31}$$

where

$$C_t^E = \phi_0 \cdot \exp\left(\phi_1\left\{1 - \frac{\theta_t K_t}{B_t}\right\}\right)\cdot Z_{t-1} \quad \text{(mimics } B_t \leq \theta_t \cdot K_t) \tag{A32}$$

$$C_t^B = \eta Z_{t-1}\left(\frac{B_t}{Y_t} - \frac{B_{ss}}{Y_{ss}}\right)^2 \tag{A33}$$

$$\theta_t \equiv \theta\left(\frac{G_t^{PD}}{\lambda_t Y_t}\right); \quad \theta'(\cdot) > 0 \tag{A34}$$

$$C_t^{EA} = \phi_0^{EA}\cdot\exp\left(\phi_1^{EA}\left\{\frac{(\lambda_t Y_t - \bar{A}\cdot Z_{g,t-1})}{G_t^{PD}} - 1\right\}\right)\cdot Z_{t-1} \tag{A35}$$

Optimality with respect to capital. Envelope and first order conditions with respect to investment and capital imply the following:

$$
\begin{aligned}
q_t &= \frac{1}{\Psi'_t} = \mathbb{E}_t\left[M_{t+1}\frac{\partial V_{t+1}}{\partial K_t}\right] - \frac{\partial C_t^E}{\partial K_t} \\
\frac{\partial V_t}{\partial K_{t-1}} &= \chi(s_t)\cdot\frac{\partial Y_t}{\partial K_{t-1}} - \frac{\partial C_t^{EA}}{\partial K_{t-1}} - \frac{\partial C_t^E}{\partial K_{t-1}} - \frac{\partial C_t^B}{\partial K_{t-1}} + q_t\left(1-\delta-\frac{\Psi'_t I_{bt}}{K_{t-1}}+\Psi_t\right).
\end{aligned}
$$

After defining

$$
R_{bt}^{GC} = \frac{\partial V_t}{\partial K_{t-1}} / \left(q_{t-1} + \frac{\partial C_{t-1}^E}{\partial K_{t-1}}\right), \tag{A36}
$$

the Euler equation associated with brown investment reads

$$
1 = \mathbb{E}_t(M_{t+1}R_{b,t+1}^{GC})
$$

C.2 Green firm

The green sector faces

$$
V_{gt} = \max_{K_{gt},I_{gt}} D_{gt} + \mathbb{E}_t(M_{t+1}V_{g,t+1}) \tag{A37}
$$

$$
D_{gt} = p_{gt}G_{gt} - (1-\tau_{gt})I_{gt} - W_{gt}L_{gt} \tag{A38}
$$

$$
K_{gt} = (1-\delta_g)K_{g,t-1} + H(I_{gt}/K_{g,t-1})K_{g,t-1} \tag{A39}
$$

First-order conditions with respect to capital and investment:

$$
q_{gt} = \mathbb{E}_t\left(M_{t+1}\frac{\partial V_{g,t+1}}{\partial K_{gt}}\right) \tag{A40}
$$

$$
q_{gt} = (1-\tau_{gt})/H'_t \tag{A41}
$$

$$
\tag{A42}
$$

The envelop condition is

$$\frac{\partial V_{gt}}{\partial K_{g,t-1}} = p_{g,t}\frac{\partial G_t^s}{\partial K_{g,t-1}} + q_{gt}\{1 - \delta_g + H_t - H_t' \cdot (I_{gt}/K_{g,t-1})\} \tag{A43}$$

After defining

$$R_{gt}^{GC} = \frac{\partial V_{gt}}{\partial K_{g,t-1}}/q_{g,t}, \tag{A44}$$

we obtain the Euler equation:

$$1 = \mathbb{E}_t(M_{t+1}R_{g,t+1}^{GC}).$$

C.3 Additional results

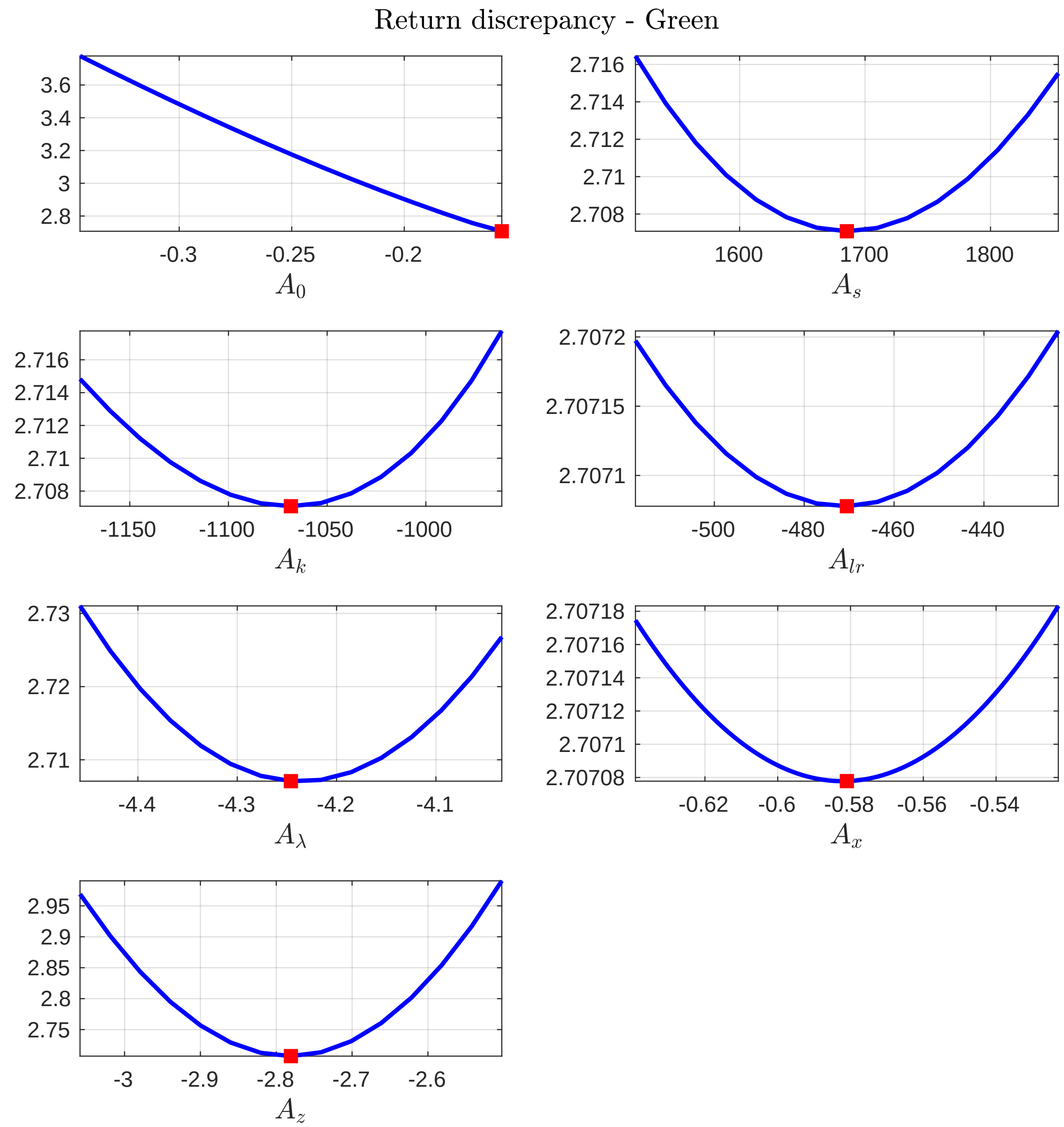

Fig. C1: Green policy (I)

Notes: This figure shows the average squared discrepancy of the returns for the brown sector:

$$\mathbb{E}\left[(R_{gt}^{P} - R_{gt}^{GC}(\theta))^2\right].$$

The markers denote the value of the parameters that minimize the objective function defined in equation (18) in main text.

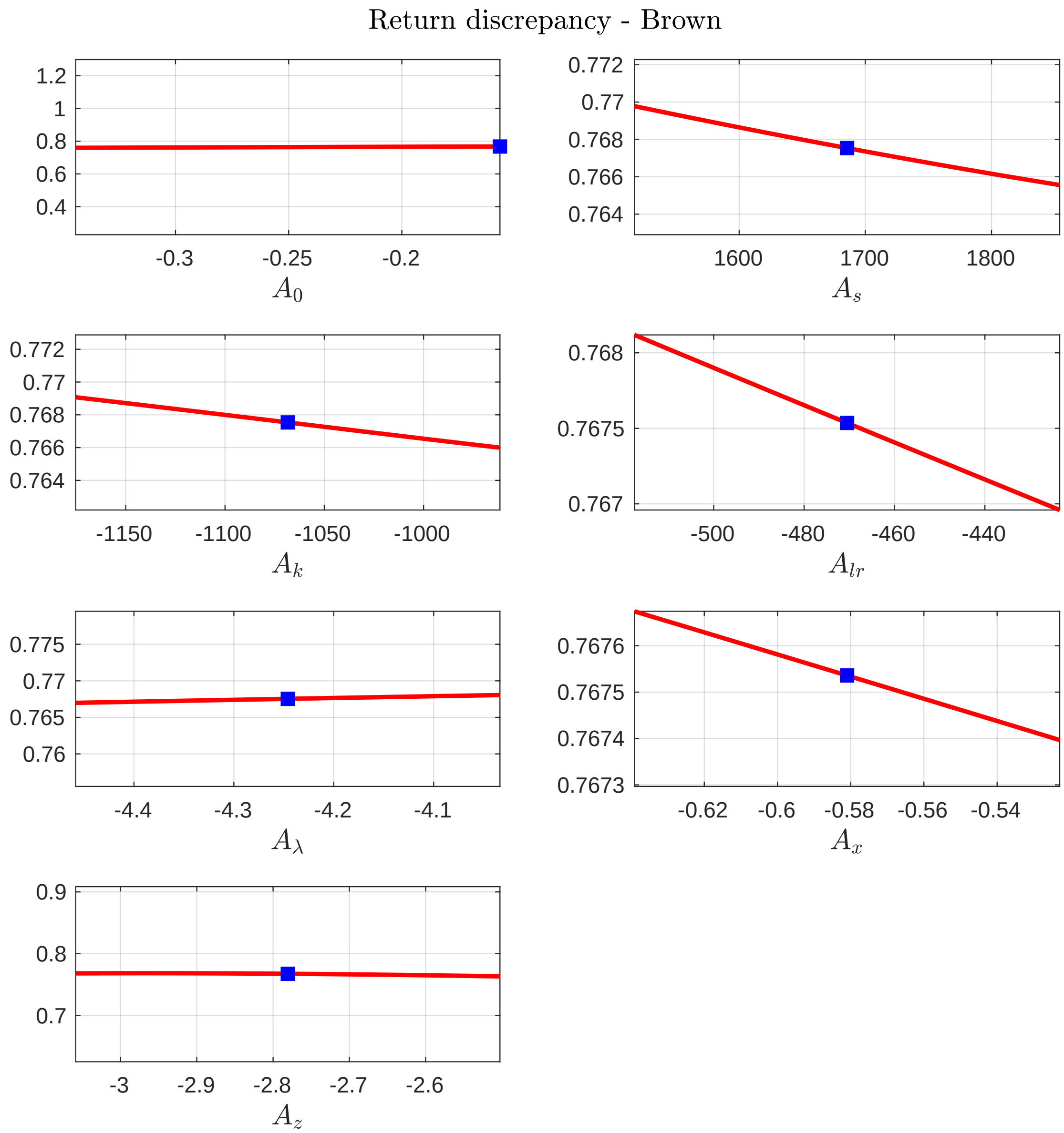

Fig. C2: Green policy (II)

Notes: This figure shows the average squared discrepancy of the returns for the brown sector:

$$\mathbb{E}\left[(R_{bt}^{P} - R_{bt}^{GC}(\theta))^2\right].$$

The markers denote the value of the parameters that minimize the objective function defined in equation (18) in main text.